

Interacting tachyon Fermi gas

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March 1, 2013

Abstract

We consider a system of many fermionic tachyons coupled to a scalar, pseudoscalar, vector and pseudovector fields. The scalar and pseudoscalar fields are responsible for the effective mass, while the pseudovector field is similar to ordinary electromagnetic field. The action of vector field ω_μ results in tachyonic dispersion relation $\varepsilon_p = \sqrt{p^2 + g^2\omega_0^2 - hpg\omega_0 - g\vec{\sigma} \cdot \nabla\omega_0 - m^2 - g\vec{\sigma} \cdot \vec{\omega}}$ that depends on helicity h and spin $\vec{\sigma}$. We apply the mean field approximation and find that there appears a vector condensate with finite average $\langle\omega_0\rangle$ depending on the tachyon density. The pressure and energy density of a many-tachyon system include the mean-field energy $\langle\varepsilon_p\rangle = \sqrt{p^2 + hpng^2/M^2 + n^2g^4/M^4 - m^2}$ which is real when the particle number density exceeds definite threshold which is $n > mM^2/g^2$ for right-handed and $n > \frac{2}{\sqrt{3}}mM^2/g^2$ for left-handed tachyons, while all tachyons are subluminal at high density. There is visible difference in the properties of right-handed and left-handed tachyons. Interaction via the vector field ω_0 may lead to stabilization of tachyon matter if its density is large enough.

1 Introduction

Tachyon is a substance whose group velocity is larger than the speed of light in vacuum $c = 1$. Tachyons are commonly known in the field theory [1] and nonlinear optics [2]. A free fermionic tachyon is described by the Lagrangian

[3, 4]

$$L = \bar{\psi} (i\gamma_5 \gamma^\mu \partial_\mu - m) \psi \quad (1)$$

corresponding to the tachyonic Dirac equation $(i\gamma^\mu \partial_\mu - \gamma_5 m)\psi = 0$ that has plane-wave solution in the form

$$\psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix} \exp(i\vec{p} \cdot \vec{r} - i\varepsilon_p t) \quad (2)$$

with a single-particle tachyon energy

$$\varepsilon_p = \sqrt{p^2 - m^2} \quad (3)$$

Much more complicated theory is developed for tachyons in external fields [5, 6]. If a charged Dirac tachyon interacts with electromagnetic field A_μ , the Lagrangian is presented in the form [7]

$$L = \bar{\psi} (i\gamma_5 \gamma^\mu D_\mu - m) \psi$$

where $D_\mu = \partial_\mu + iA_\mu$. Dirac tachyons with nonlinear self-interaction were also considered [8]. However a system of many interacting tachyons has not been discussed yet.

In the present paper we consider the tachyonic Lagrangian

$$L = \bar{\psi} (i\gamma_5 \gamma^\mu \partial_\mu - m + s + i\gamma_5 \Pi - g\gamma^\mu \omega_\mu - \gamma_5 \gamma^\mu A_\mu) \psi + \frac{1}{2} (\partial^\nu s \partial_\nu s - M_s^2 s^2) + \frac{1}{2} (\partial^\nu \Pi \partial_\nu \Pi - M_\pi^2 \Pi^2) + \frac{1}{2} (G^{\mu\nu} G_{\mu\nu} + M^2 \omega_\mu^2) + \frac{1}{2} (F^{\mu\nu} F_{\mu\nu} + M_A^2 A_\mu^2) \quad (4)$$

that includes interaction with scalar field s , pseudoscalar field Π , massive vector field ω_μ (relevant Maxwell tensor is $G_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$) and massive electromagnetic field A_μ ($F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$). Each interaction vertex in (4) is taken in the form of Yukawa minimal coupling while the coupling constants are included in the fields for simplicity. Our interest concerns the single-particle energy spectrum and the thermodynamical functions of many tachyon system in the frames of the mean-field approach. The interaction may result in a field condensate with non-zero vacuum expectation value responsible for qualitatively new properties in contrast to a free tachyon Fermi gas.

2 Tachyonic energy spectrum

Starting with the Lagrangian (4), we write the equation of motion for the Dirac tachyon

$$(i\gamma^\mu \partial_\mu - \gamma_5 m + \gamma_5 s + i\Pi - \gamma_5 \gamma^\mu \omega_\mu - \gamma^\mu A_\mu) \psi = 0 \quad (5)$$

and Klein-Gordon equations

$$\partial^\mu \partial_\mu s + M_s^2 s = \bar{\psi} \psi \quad (6)$$

$$\partial^\mu \partial_\mu \Pi + M_\pi^2 \Pi = \bar{\psi} \gamma_5 \psi \quad (7)$$

$$\partial^\mu G_{\mu\nu} + M^2 \omega_\mu = \bar{\psi} \gamma_\mu \psi \quad (8)$$

$$\partial^\mu F_{\mu\nu} + M_A^2 A_\mu = \bar{\psi} \gamma_\mu \gamma_5 \psi \quad (9)$$

for the fields s , Π , ω_μ and A_μ , where the tachyons are acting as sources.

Looking for solution of equation (5) in the form of plane wave (2), we come to a linear system for bispinors

$$\begin{cases} \vec{\sigma} \cdot (\vec{p} - \vec{A}) + m - s - \omega_0 \\ \vec{\sigma} \cdot (\vec{p} - \vec{A}) - m + s - \omega_0 \end{cases} \chi = (\varepsilon_p + \vec{\sigma} \cdot \vec{\omega} + A_0 + i\Pi) \phi \quad (10)$$

It has solution if the energy satisfies dispersion relation

$$(\varepsilon_p + \vec{\sigma} \cdot \vec{\omega} + A_0)^2 = \left[\vec{\sigma} \cdot (\vec{p} - \vec{A}) - \omega_0 \right]^2 - m_*^2 \quad (11)$$

where

$$m_*^2 = (m - s)^2 + \Pi^2 \quad (12)$$

is the effective mass. Indeed, switching off all interactions, we obtain from (11) the energy spectrum of a free tachyon (3).

Taking into account the properties of the Dirac matrices γ_μ and operator $\vec{p} = -i\nabla$, we find that the energy spectrum (11) at $\omega_\mu = 0$ is reduced to

$$\varepsilon_p = \sqrt{\left[\vec{\sigma} \cdot (\vec{p} - e\vec{A}) \right]^2 - m_*^2 - eA_0} = \sqrt{\left(\vec{p} - e\vec{A} \right)^2 - e\vec{\sigma} \cdot \text{rot}\vec{A} - m_*^2 - eA_0} \quad (13)$$

where the coupling constant e is written explicitly so that expression (13) is looking like well known energy spectrum in electrodynamics and nuclear

physics [9] where potential A_μ is acting as an ordinary electromagnetic or vector-meson field, while the scalar field s and pseudoscalar field Π are responsible for the effective mass [12].

When the vector field $\omega_\mu \neq 0$ is switched on, an absolutely different tachyonic energy spectrum is obtained:

$$\varepsilon_p = \sqrt{p^2 + g^2\omega_0^2 - g\omega_0\vec{\sigma} \cdot \vec{p} - g\vec{\sigma} \cdot \nabla\omega_0 - m^2 - g\vec{\sigma} \cdot \vec{\omega}} \quad (14)$$

Here we omit for simplicity all other fields (s , Π , A_μ) and write the coupling constant g explicitly. There is not only dependence on the spin (note that $\nabla\omega_0$ is acting as a regular magnetic field applied to an ordinary non-tachyon particle) there is also explicit dependence on the helicity of the particle.

For uniform electro-static potential $\omega_0 = \text{const}$ (and $\vec{\omega} = 0$) formula (14) is reduced to

$$\varepsilon_p = \sqrt{p^2 + g^2\omega_0^2 - hg\omega_0p - m^2} \quad (15)$$

where $h = 1$ for right-handed and $h = -1$ left-handed tachyons, respectively. The energy (15) of right-handed tachyons is imaginary at any p if

$$-m < g\omega_0 < \frac{2}{\sqrt{3}}m \quad (16)$$

The energy (15) of left-handed tachyons is imaginary at any p if

$$-\frac{2}{\sqrt{3}}m < g\omega_0 < m \quad (17)$$

The group velocity

$$v = \frac{d\varepsilon_p}{dp} = \frac{p - \frac{1}{2}hg\omega_0}{\sqrt{p^2 + g^2\omega_0^2 - hg\omega_0p - m^2}} \quad (18)$$

is subluminal at any p if

$$|g\omega_0| > \frac{2}{\sqrt{3}}m \quad (19)$$

3 Mean field approach to many-tachyon system

Now consider a many-tachyon system with interaction described by the Lagrangian (4) and enclosed in volume $V = \int d^3r$. A charge Q , associated with

the relevant Noether current, is incorporated in the partition function [9]

$$Z = \int [d\bar{\psi}] [d\psi] \exp \left\{ \int_0^{1/T} d\tau \left(\int L d^3r + \mu Q \right) \right\} \quad (20)$$

where $\tau = it$, and μ is the tachyonic chemical potential. The tachyonic equation of motion (5) results in the continuity equation $\partial_\mu j_5^\mu = 0$ for the axial current $j_5^\mu = \bar{\psi} \gamma^\mu \gamma_5 \psi$ that corresponds to the conserved axial charge $Q_5 = \int j_5^0 d^3r$, while the vector current $j^\mu = \bar{\psi} \gamma^\mu \psi$ is not conserved because [10] $\partial_\mu j^\mu = -2im\bar{\psi} \gamma_5 \psi$. Hence, the axial charge is incorporated in the partition function (20) rather than the number of particles (which is not conserved). Thus, the tachyonic pressure

$$P = T \ln Z \quad (21)$$

the energy density

$$E = \mu n_5 + TS - P \quad (22)$$

and the entropy density

$$S = \left(\frac{\partial P}{\partial T} \right)_{V, \mu} \quad (23)$$

include the axial charge density

$$n_5 = T \frac{\partial (\ln Z)_{V, T}}{\partial \mu} \quad (24)$$

rather than ordinary particle number density.

Let us apply the mean field approximation [9], neglecting quantum field fluctuations. The tachyons are taken to move independently in the presence of constant mean fields $\langle s \rangle$, $\langle \Pi \rangle$, $\langle \omega_\mu \rangle$ and $\langle A_\mu \rangle$ which themselves are generated self-consistently according to equations (6)-(9), namely:

$$M_s^2 \langle s \rangle = \langle \bar{\psi} \psi \rangle \quad (25)$$

$$M_\pi^2 \langle \Pi \rangle = \langle \bar{\psi} \gamma_5 \psi \rangle \quad (26)$$

$$M^2 \langle \omega_\mu \rangle = \langle \bar{\psi} \gamma_\mu \psi \rangle \quad (27)$$

$$M_A^2 \langle A_\mu \rangle = \langle \bar{\psi} \gamma_\mu \gamma_5 \psi \rangle \quad (28)$$

The argument of partition function (20) is

$$\bar{L} + \tilde{L} + \mu Q_5$$

where \bar{L} is the Lagrangian of tachyons and

$$\tilde{L} = -\frac{1}{2}M_s^2 \langle s \rangle^2 - \frac{1}{2}M_\pi^2 \langle \Pi \rangle^2 + \frac{1}{2}M^2 \langle \omega_\mu \rangle^2 + \frac{1}{2}M^2 \langle A_\mu \rangle^2$$

is the Lagrangian of the mean fields. It allows to present the partition function (20) as a product $Z \cong \langle Z \rangle = \bar{Z} \tilde{Z}$ that includes contribution of the tachyons \bar{Z} and contribution of the mean fields \tilde{Z} . Thus, we determine the pressure

$$\langle P \rangle = \bar{P} - \frac{1}{2}M_s^2 \langle s \rangle^2 - \frac{1}{2}M_\pi^2 \langle \Pi \rangle^2 + \frac{1}{2}M^2 \langle \omega_\mu \rangle^2 + \frac{1}{2}M^2 \langle A_\mu \rangle^2 \quad (29)$$

and the energy density

$$\langle E \rangle = \bar{E} + T \left[\left(\frac{\partial \langle P \rangle}{\partial T} \right)_{V,\mu} - \left(\frac{\partial \bar{P}}{\partial T} \right)_{V,\mu} \right] + \frac{1}{2}M_s^2 \langle s \rangle^2 + \frac{1}{2}M_\pi^2 \langle \Pi \rangle^2 - \frac{1}{2}M^2 \langle \omega_\mu \rangle^2 - \frac{1}{2}M^2 \langle A_\mu \rangle^2 \quad (30)$$

where [11, 12]

$$\bar{P} = \frac{\gamma}{2\pi^2} T \int_0^\infty \ln \left(1 + \exp \frac{\mu - \langle \varepsilon_p \rangle}{T} \right) p^2 dp \quad (31)$$

is the pressure, while

$$\bar{E} \equiv \mu n_5 + T \left(\frac{\partial \bar{P}}{\partial T} \right)_{V,\mu} - \bar{P} = \frac{\gamma}{2\pi^2} \int_0^\infty \langle \varepsilon_p \rangle f_\varepsilon p^2 dp \quad (32)$$

is the energy density and

$$n_5 = \frac{\gamma}{2\pi^2} \int_0^\infty f_\varepsilon p^2 dp \quad (33)$$

is the axial charge density of a free tachyon Fermi gas with distribution function

$$f_\varepsilon = \frac{1}{\exp [(\langle \varepsilon_p \rangle - \mu) / T] + 1} \quad (34)$$

and single-particle energy spectrum $\langle \varepsilon_p \rangle$ determined in the frames of mean-field approximation by substituting the mean fields $\langle s \rangle$, $\langle \Pi \rangle$, $\langle \omega_\mu \rangle$ and $\langle A_\mu \rangle$ in (11).

4 Vector and pseudovector condensates

Let us consider tachyons with pure vector and pseudovector interaction ω_μ and A_μ . For a rotational symmetric uniform matter we take $\langle \vec{\omega} \rangle = \langle \vec{A} \rangle = 0$, $\langle \omega_0 \rangle = \tilde{\omega}_0 = \text{const}$ and $\langle A_0 \rangle = \tilde{A}_0 = \text{const}$ so that equations (28) and (27) yield

$$\tilde{\omega}_0 = -\frac{g}{M^2}n \quad (35)$$

$$\tilde{A}_0 = -\frac{e}{M^2}n_5 \quad (36)$$

where, again, the coupling constants g and e are written explicitly, and

$$n_5 = \langle \bar{\psi} \gamma^0 \gamma_5 \psi \rangle \quad (37)$$

is the axial charge density, while

$$n = \langle \bar{\psi} \gamma^0 \psi \rangle \quad (38)$$

is the particle number density (and the coupling constants g and e are written explicitly).

Thus, the vector and pseudovector fields acquires finite condensed values \tilde{A}_0 and $\tilde{\omega}_0$ which depends on n . Substituting $\tilde{\omega}_0$ (35) in (14) we find the mean-field energy spectrum of tachyons

$$\langle \varepsilon_p \rangle = \sqrt{p^2 + \frac{n^2 g^4}{M^4} + hp \frac{ng^2}{M^2} - m^2 - eA_0} \quad (39)$$

Defining effective single-particle energy

$$\varepsilon_{*p} = \sqrt{p^2 + \frac{n^2 g^4}{M^4} + hp \frac{ng^2}{M^2} - m^2} \quad (40)$$

together with effective chemical potential

$$\mu_* = \mu + eA_0 \quad (41)$$

and taking into account that the distribution function (34) can be presented in the form

$$f_\varepsilon \equiv \frac{1}{\exp[(\varepsilon_{*p} - \mu_*)/T] + 1} \quad (42)$$

we calculate the pressure (29) and energy density (30) so

$$\langle P \rangle = \bar{P} + \frac{1}{2} \frac{g^2 n^2}{M^2} + \frac{1}{2} \frac{e^2 n_5^2}{M_A^2} \quad (43)$$

$$\langle E \rangle = E_* + T \left[\frac{e^2 n_5}{M_A^2} \left(\frac{\partial n_5}{\partial T} \right)_{v,\mu} - \frac{g^2 n}{M^2} \left(\frac{\partial n}{\partial T} \right)_{v,\mu} \right] - \frac{1}{2} \frac{g^2 n^2}{M^2} + \frac{1}{2} \frac{e^2 n_5^2}{M_A^2} \quad (44)$$

where

$$E_* = \frac{\gamma}{2\pi^2} \int_0^\infty \varepsilon_{*p} f_\varepsilon p^2 dp \quad (45)$$

is the energy density of free particles with energy spectrum ε_{*p} (49) and chemical potential (41).

The single-particle energy (49) is given in Fig. 1 for right-handed and left-handed tachyons. The single-particle energy (49) of right-handed tachyons ($h = 1$) is always real at any p when

$$n > n_+ = \frac{mM^2}{g^2} \quad (46)$$

while the single-particle energy of left-handed tachyons ($h = -1$) is always real at any p when

$$n > n_- = \frac{2}{\sqrt{3}} \frac{mM^2}{g^2} \quad (47)$$

Thus, the pressure (29), (43) and energy density (30), (44) acquire no imaginary content if the particle number density exceeds critical threshold n_+ (46) or n_- (47).

The group velocity

$$v = \frac{d\varepsilon_{*p}}{dp} = \frac{p \pm ng^2/(2M^2)}{\sqrt{p^2 + n^2g^4/M^4 \pm png^2/M^2 - m^2}} \quad (48)$$

associated with energy-spectrum (49), is subluminal for both right-handed and left-handed tachyons when inequality (47) is satisfied. This group velocity is plotted in Fig. 2.

What is important for us is be noted than in the limit $m \rightarrow 0$ the energy spectrum () is

$$\varepsilon_{*p} = \sqrt{p^2 + \frac{n^2g^4}{M^4}} \pm p \frac{ng^2}{M^2} \quad (49)$$

and the group velocity v is helicity dependent

$$v = \frac{p \pm ng^2 / (2M^2)}{\sqrt{p^2 + n^2g^4/M^4 \pm png^2/M^2}} \quad (50)$$

and It is given in Fig. 3 and Fig. 4. Omega is included in the mass matrix, so that the vector condensate results in the mass generation.

5 Conclusion

The energy dispersion relation (11) of a tachyon coupled to the scalar, pseudoscalar, vector and pseudovector (electromagnetic) fields reveal the nature of interaction. The scalar and pseudoscalar potentials field contribute to the only effective mass (12). The potential is acting as a regular electromagnetic field on the ordinary Dirac particle. The most interesting behavior belongs to the vector field, whose sole action results in very peculiar energy spectrum (14) including dependence not only on the spin but also on the helicity of the tachyon.

Working in the frames of mean-field approximation, we find that the single-particle energy (39), (Fig. 1) is real under conditions (46)-(47) that imply that the pressure (43) and energy density (44) are also real like that of an ordinary Fermi gas. This means that interaction results in stabilization of the tachyon matter, and weak coupling ($g \rightarrow 0$) can provide stability to the dense matter, while strong coupling ($g \rightarrow \infty$) is enough for stabilization of rarefied matter. It should be also noted that the group velocity (50) becomes subluminal in dense tachyon Fermi gas, see Fig. 2, although right-handed and left-handed tachyons reveal very different behavior: for example, left handed tachyons may have negative group velocity like the optical tachyons in nonlinear medium [2], and subluminal right-handed tachyons are faster than left-handed tachyons at the same momentum.

The present analysis may give further ideas for solving applied problems. For example, we have all necessary theoretical background (10)-(11), (20) to consider superconductivity of interacting tachyon Fermi system and find the relevant excitation spectrum. It should be emphasized that conditions (46)-(47) do not warrant automatically the causality condition [11], which may impose further requirements to the particle number density. According to (44) the vector and pseudovector (electromagnetic) interaction bring

different repulsive-attractive effects to the many-particle system, and formulas (29)-(34), (39)-(45) can be applied to calculation of the binding energy of tachyon matter and finding the point of gas-liquid phase transition. Solving such problems, one should be careful with expression of the Fermi momentum, bearing in mind the important fact that formulas (39)-(50) are written with the particle number density $n = \langle \bar{\psi} \gamma^0 \psi \rangle$, while the partition function of a free tachyon Fermi gas must include the axial charge density $n_5 = \langle \bar{\psi} \gamma^0 \gamma_5 \psi \rangle$.

The author is grateful to Erwin Schmidt for inspiring discussions.

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Figure 1: Single-particle tachyon energy at small density (dotted), critical density (46) or (47) (dashed) and large density (solid) vs momentum (both in the unit of mass m) for right-handed (R) and left-handed (L) tachyons.

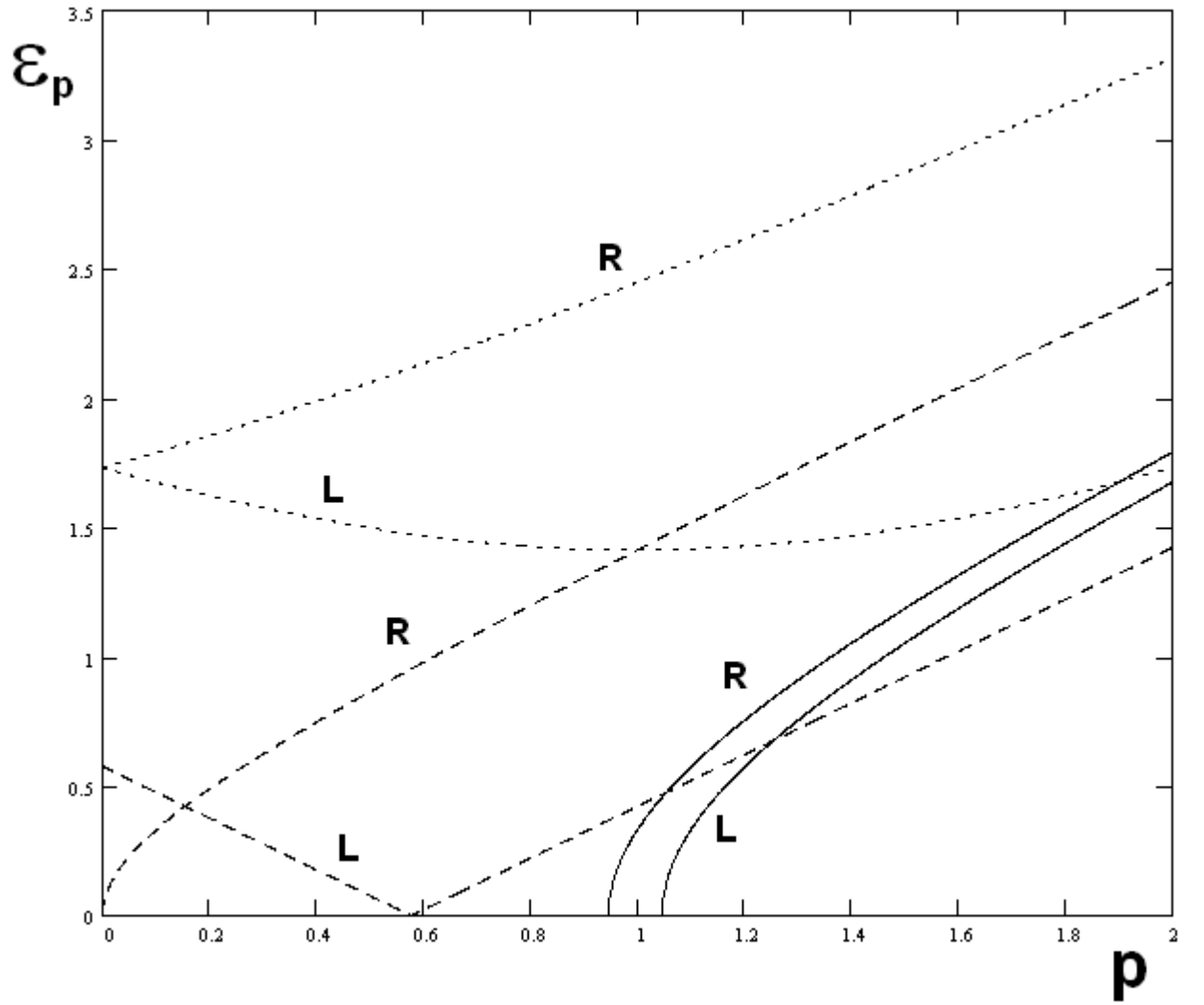


Figure 2: Group velocity of right-handed (R) and left-handed (L) tachyons at small density (dotted), critical density (46) or (47) (dashed) and large density (solid).

